

## Error Level Fusion of Multimodal Biometrics

Madasu Hanmandlu<sup>a</sup>, Jyotsana Grover<sup>a</sup>,  
Shantaram Vasirkala<sup>b</sup>, Hari Mohan Gupta<sup>a</sup>

<sup>a</sup>Electrical Engineering Department, Indian Institute of Technology, Delhi, New Delhi-110016, India

<sup>b</sup>Computer Science Department, California State University, Dominguez Hills Carson, CA 90747

### Abstract

This paper presents a multimodal biometric system based on error level fusion. Two error level fusion strategies, one involving the Choquet integral and another involving the t-norms are proposed. The first strategy fully exploits the non additive aspect of the integral that accounts for the dependence or the overlapping information between the error rates FAR's and FRR's of each biometric modality under consideration. A hybrid learning algorithm using combination of Particle Swarm Optimization, Bacterial Foraging and Reinforcement learning is developed to learn the fuzzy densities and the interaction factor. The second strategy employs t-norms that require no learning. The fusion of the error rates using t-norms is not only fast but results in very good performance. This sort of fusion is a kind of decision level fusion as the error rates are derived from the decisions made on individual modalities. The experimental evaluation on two hand based datasets and two publically available datasets confirms the utility of the error level fusion.

*Keywords:* Error level fusion, Multimodal hand based Biometrics, T-norms, Error Rates, Choquet integral.

### 1. Introduction

Unimodal Biometric systems have several limitations such as susceptibility of the authentication results to the quality of the samples, orientation/rotation and distortion of the sample, noise, intra-class variability, non-distinctiveness, non-universality, risk of spoofing, and others. Some of the limitations imposed by the unimodal biometric systems for establishing the identity can be overcome by resorting to multiple biometric modalities, which utilize the evidences presented by the biometric sources to determine or confirm the identity of an individual. Information from multiple sources can be integrated at four distinct levels, viz., sensor level (combining the raw data acquired from the sensors), feature level (integrating the features of different biometrics), score level (aggregating the genuine and imposter scores) and decision level (combining the decisions).

Fusion at the feature level involves combining the features of different modalities before the classification phase [5, 6, 7, 8, 9, 10]. Although the features are rich containing raw source of information, feature level fusion has some drawbacks. Features from different modalities may not be compatible, thus making the fusion arduous (for example some features might be binary and some might be real). Moreover feature level is an uphill task as large dimensionality of the feature space may pose problems at the classification stage.

Score level fusion, widely studied in the literature can be tackled in three ways. The first one [11] employs the scalar functions such as sum, product, mean, max, min, exponential rules etc. Here the scores of each of the modalities must be transformed to the common domain. The second one aggregates the scores by using the likelihood ratio statistic [14, 20] computed using the generalized densities estimated from the genuine and imposter scores. The third one carries out the score level fusion by a classifier. In this, a feature vector is constructed from the matching scores of individual modalities and it is then classified into

either genuine or imposter. Some of the familiar classification approaches in vogue for the score level fusion are SVM [17, 18], Neural Networks [19] and fuzzy logic. In the first category of approaches, score normalization is imperative, but in the second and third categories score normalization can dramatically enhance the performance of an authentication system.

Decision level is less studied and is considered inferior to score-level fusion as the decisions have less information content as compared to "soft" matching scores. One approach for decision level fusion is by the majority vote [21], which counts the number of decisions from the component classifiers, and chooses the majority of the decision as the final decision. Its derivative called weighted majority voting [42], assigns different weights to different classifiers as per the performance of the component classifiers. This approach transforms the output values in the form of binary decisions to the continuous numbers. Other approaches include: Bayesian decision fusion [23], the Dempster-Shafer theory of evidence [24], all of which convert the decisions into scores, with the converting parameters learned from a training set.

In yet another approach [1], the authentication results from different modalities are combined by using fuzzy k-means (FKM) and fuzzy vector quantization (FVQ) algorithms, and median radial basis function (MRBF) network. Here FKM and FVQ are modified based on the fuzzy vector distance. Two different strategies are investigated in [2] for the decision level fusion. In the first strategy an optimal fusion rule involving the likelihood ratio test (LRT) is selected and the Chair Varshney constraint is applied for the correlated hypothesis testing where the thresholds of the individual biometric classifiers are fixed. In the second strategy, a particle swarm optimization (PSO) based procedure [30] is investigated for simultaneously optimizing the thresholds and the fusion rule. But the problem here is that the number of rules increases exponentially with the increase in the number of modalities. A non parametric statistical approach is presented in [36] for the decision level fusion, which stresses upon the importance of classifier selection while combining the decisions. However this technique requires sufficient training data to obtain reasonable estimates of the densities of the classifier output. An optimal fusion scheme is proposed in [3] for the decision level fusion using the AND rule or OR rule by optimizing the matching score thresholds. In [33], feedforward network model is used to generate different combinations of the hyperbolic functions to achieve decision level fusion. It is proved by Daugman [25] that the combination of the two decisions using AND and OR rules often has the risk of degrading the overall performance when the performance of component classifiers is significantly different. In [40], the estimates for posterior probabilities are considered independent and identically distributed (normal or uniform) and the classification error is calculated using six fusion methods: average, minimum, maximum, median, majority vote, and oracle. A comparative study of four integrals: Choquet, Sugeno, Upper and lower integrals is made in [41] for the decision level fusion whereas t-norms are used on the Biometric database created at IIT Delhi in [45].

The proposed error level fusion employs the Choquet integral and t-norms on four databases, the first comprising three modalities, viz., hand veins, hand geometry and palmprint and the second comprising palmprint, left hand index finger knuckle and left hand middle finger knuckle, the third one is the XM2VTS database, and the fourth one is the NIST database. Error level is also a kind of decision level fusion wherein the error rates assume the role of decisions. The motivation for the proposed work is two-fold: To exploit the less known hand based modalities and to explore two different approaches for the error level fusion. The hand possesses a rich repository of biometric information, which has not

been tapped to the fullest extent except the fingerprint. This work endeavors to tap this potentiality by way of hybrid learning.

The rest of paper is organized as follows: Section 2 presents two approaches for the error level fusion: one by the Choquet integral and another by the t-norms. Section 3 introduces a new hybrid learning method. Section 4 briefs the results of experimentation. The conclusions are given in Section 5.

## 2. Error level fusion

The proposed error level fusion is devised using two methodologies that permit us to fuse the errors of any number of modalities. It deals with fusion of error rates, i.e. the aggregation of error rates, FAR (false acceptance rate), FRR (false rejection rate) obtained as the outcome of classifiers of individual modalities. The error rates can be considered as probabilities denoted by:

$$FAR = P(u_f = 1/H_0), \quad FRR = P(u_f = 0/H_1) \quad (1)$$

where  $H_0$  is the null hypothesis representing the imposter scores and  $H_1$  is the true hypothesis representing the genuine scores; the decision of  $u_f = 1$  indicates that the user is genuine and that  $u_f = 0$  represents that user is an imposter. The genuine acceptance rate is given by  $GAR = 1 - FRR$ .

The performance of a biometric system can be judged by a plot of GAR vs. FAR called Receiver Operating Characteristic (ROC). We vary the threshold to obtain the genuine scores arrived at by matching the features of the same user's samples and impostor scores arrived at by matching the features of different users' samples. A pair of (GAR, FAR) serves as an operating point on ROC, which corresponds to a particular threshold T of the matching scores. The decision of whether a person is genuine or imposter is made by the likelihood test [47]

$$\frac{p(u_f = 0/H_1)}{p(u_f = 1/H_0)} \underset{u_g=0}{\overset{u_g=1}{\gtrless}} T \quad (2)$$

where  $u_g = 1$  if the ratio is greater than T and  $u_g = 0$ , if the ratio is less than T.

In the context of the error level fusion, the fuzzy sets formed from the error rates of the modalities can be combined non- additively by constructing the fuzzy rules and then aggregating them using a fuzzy integral. This approach has an accompanying problem of representing the fuzzy sets with the membership functions and computing the rule strengths. This problem is avoided by normalizing the error rates and then combining them through a fuzzy integral or t-norms. However the use of fuzzy integral is facilitated only by the fuzzy measures that account for the interaction or the overlapping information among the error rates of the different modalities. The main effort in this work is directed towards cashing in on the interactions among the error rates.

### 2.1 Properties of Fuzzy Integral

Before embarking upon the error level fusion by a fuzzy integral some background on Fuzzy integrals needs to be built up. As the fuzzy measures [4] are involved in the evaluation of a fuzzy integral, a few properties are necessitated here. A fuzzy measure g over a set X (the universe of discourse with the subsets A, B) satisfies the following conditions:

$$1. \text{ When } A = \emptyset \text{ then } g(A) = 0 \quad (3)$$

$$2. \text{ When } A \subseteq B \text{ then } g(A) \leq g(B) \quad (4)$$

Let  $X = \{x_1, x_2, \dots, x_n\}$  now stand for a finite set of error rates and  $\lambda \in (-1, +\infty)$  be the Sugeno's  $\lambda$ -measure that accounts for the interaction between its two subsets  $A$  and  $B$ . The  $\lambda$ -measure is a function  $g$  from  $2^X$  to  $[0, 1]$  with the additional properties:

3.  $g(X) = 1$
4. if  $A, B \subseteq 2^X$  with  $A \cap B = \emptyset$  then
 
$$g(A \cup B) = g(A) + g(B) + \lambda g(A)g(B) \quad (5)$$

As a convention, the value of  $g$  for a singleton set  $\{x_i\}$  is called the fuzzy density denoted by  $g^i = g(\{x_i\})$ . Recursive applications of Eq. (5) till all the error rates are covered will result in a  $(n - 1)$  degree polynomial equation in  $\lambda$ :

$$\lambda + 1 = \prod_{i=1}^n (1 + \lambda g^i), \quad \lambda \neq 0, \quad \lambda > -1 \quad (6)$$

Solving the above equation is computationally expensive, and to simplify this problem a new class of fuzzy measures called  $q$ -measures is formulated in [35].

### 2.2 Q-measures

We now prepare the ground for the definition of  $q$ -measure. Given  $X$  with  $A, B \subseteq X$  and  $A \cap B = \emptyset$ , a set function  $g : 2^X \rightarrow [0, \infty)$  must fulfill the conditions to be qualified as a fuzzy measure:

$$g(\Phi) = 0 \quad (7)$$

$$g(A \cup B) = g(A) + g(B) + \lambda g(A)g(B), \lambda > -1 \quad (8)$$

The fuzzy densities  $g^i$  satisfy the two constraints:

$$0 \leq g^i \leq 1, \quad i = 1, 2, 3, \dots, n \quad (9)$$

$$\sum_{i=1}^n g^i > 0 \quad (10)$$

The above two constraints force the fuzzy densities to have values in the interval  $[0,1]$  for any  $\lambda > -1$ . Since  $g$  satisfies the monotonicity axiom in Eq.(4) in the sense of set inclusion,  $g(X)$  is guaranteed to have the maximum value among the fuzzy measures of all possible subsets, i.e.  $\forall A \subseteq X, g(A) \leq g(X)$ .

Given a set of fuzzy densities  $\{g^1, g^2, \dots, g^n\}$ , that satisfies the constraints in Eq.(9) and Eq.(10), the  $q$ -measure  $q : 2^X \rightarrow [0, 1]$  is defined as:

$$q(A) = \frac{g(A)}{g(X)}, \quad \forall A \subseteq X \quad (11)$$

For any choice of the variable  $\lambda > -1$ , the  $q$ -measure can be constructed using Eq.(11). The  $\lambda$ -measure is a special case of  $q$ -measure with  $g(X) = 1$ ; so  $q$ -measure doesn't require the solution of Eq.(6).

### 2.3 Finding the q-measures from the fuzzy densities

Firstly the interaction factor  $\lambda$  is initialized. Considering that all the fuzzy densities are equally important, they are initialized as:

$$g^i = \frac{1}{n} \quad (12)$$

Next these densities are used to compute the fuzzy measures of the sets  $\{x_{(i)}, \dots, x_{(n)}\}$ , for  $i = n - 1, n - 2, \dots, 3, 2, 1$  recursively from:

$$\begin{aligned} g(\{x_{(i)}\}) &= g^i \\ g\{x_{(i)}, \dots, x_{(n)}\} &= g^i + g\{x_{(i+1)}, \dots, x_{(n)}\} + \lambda g^i g\{x_{(i+1)}, \dots, x_{(n)}\} \end{aligned} \quad (13)$$

After computing the fuzzy measures using Eq.(13),  $q$ -measures follow from Eq.(11). These fuzzy measures are also needed in the Choquet integral. The required fuzzy densities and  $\lambda$  are updated using the Reinforced BF-PSO hybrid learning technique discussed later in Section 3.

#### 2.4 The Choquet Integral

A fuzzy integral is used for the error level fusion wherein the fuzzy densities indicate the importance of the error rates of individual biometric modality whereas the fuzzy measures give the interaction among the error rates of all biometric modalities concerned. Since the fuzzy densities represent the relative importance of each of the error rates in the error level fusion, the order in which the error rates should be combined to yield the desired accuracy is decided by the information sources.

A fuzzy integral aggregates a real function ( $h$ ) serving as the information source with respect to the  $q$ -measure ( $q$ ) that provides the confidence over  $h$ . Let  $h : X \rightarrow [0, 1]$  be a real function and the set  $A_i = \{x_{(i)}, x_{(i+1)}, x_{(i+2)}, \dots, x_{(n)}\}$  is derived from  $X$ , then the Choquet integral [26, 27, 28] is expressed as:

$$C(h) = \sum_{i=1}^n \left[ h(x_{(i)}) - h(x_{(i-1)}) \right] q(A_i) \quad (14)$$

where the notation,  $(i)$ , i.e. The bracketed subscript means that the indices are permuted so that  $h(x_{(1)}) \leq h(x_{(2)}) \leq \dots \leq h(x_{(n)})$ ,  $h(x_{(0)}) = 0$ .

#### 2.5 Error level fusion using the Choquet Integral

The error rates are considered as the information sources in the Choquet Integral. Accordingly, we need one pair of error rates ( $FAR_{ij}, FRR_{ij}$ ) corresponding to  $i^{th}$  modality and  $j^{th}$  threshold.  $FAR_{1j}, FAR_{2j}$ , and  $FAR_{3j}$  are the false acceptance rates of three modalities and  $FRR_{1j}, FRR_{2j}$ , and  $FRR_{3j}$  are the corresponding false rejection rates. These error rates are normalized to lie in the range  $[0, 1]$  by the relations:

$$FAR_{ij} = \frac{FAR_{ij} - \min_j(FAR_{ij})}{\max_j(FAR_{ij}) - \min_j(FAR_{ij})}, \quad i = 1, 2, 3 \quad j = 1 \text{ to } p \quad (15)$$

$$FRR_{ij} = \frac{FRR_{ij} - \min_j(FRR_{ij})}{\max_j(FRR_{ij}) - \min_j(FRR_{ij})}, \quad i = 1, 2, 3 \quad j = 1 \text{ to } p \quad (16)$$

where  $p$  is the number of thresholds. The information sources are then permuted so that  $h(x_{(1)}) = FAR_{(1)j} \leq h(x_{(2)}) = FAR_{(2)j} \leq h(x_{(3)}) = FAR_{(3)j}$ . The aggregated False acceptance rate due to  $j^{th}$  threshold is given by :

$$AFAR_j = \sum_{i=1}^3 \left[ FAR_{(i)j} - FAR_{(i-1)j} \right] q(A_i) \quad (17)$$

where  $A_i = \{FAR_{(i)j}, FAR_{(i+1)j}, \dots, FAR_{(3)j}\}$  and  $q(A_i)$  represents the  $q$ -measure of the set  $A_i$ . Similarly, the aggregated False Rejection rate due to  $j^{th}$  threshold is given by:

$$AFRR_j = \sum_{i=1}^3 [FRR_{(i)j} - FRR_{(i-1)j}]q(B_i) \quad (18)$$

where  $B_i = \{FRR_{(i)j}, FRR_{(i+1)j}, \dots, FRR_{(3)j}\}$  and  $q(B_i)$  represents the  $q$ -measure of the set  $B_i$ .

The goal is to get the minimum aggregated errors  $AFAR_j$  and  $AFRR_j$  corresponding to  $j^{th}$  threshold. These aggregated errors serve as the criterion (minimization) function in the Reinforced BF-PSO hybrid learning. The fuzzy measures and the interaction factor need to be learnt using this hybrid learning. Fig.1 shows the schematic diagram of the error level fusion using Choquet integral.

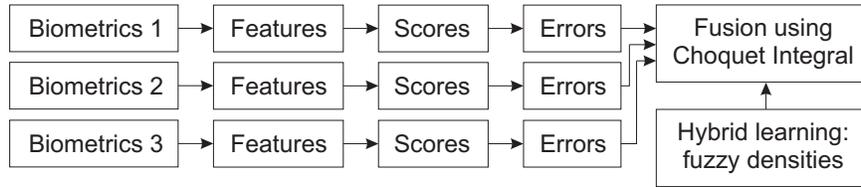


Fig. 1: Error level fusion using Choquet Integral

### 2.6 Error level Fusion using t-norms

Before delving on the use of t-norms for the error level fusion, a few preliminaries of t-norms are in order. The use of these norms to the error level fusion is not yet undertaken in the literature. The relevance of t-norms for the fusion is explored in this work. Triangular norms [32, 34] (t-norms) and t-conorms are the most general families of binary functions that satisfy the requirements of the conjunction and disjunction operators, respectively. These t-norms  $T(x, y)$  and t-conorms  $S(x, y)$  are two-place functions that map the unit square into the unit interval, i.e.  $T(x, y):[0,1] \times [0,1] \rightarrow [0,1]$  and  $S(x, y):[0,1] \times [0,1] \rightarrow [0,1]$ . Specifically, t-norms do not require the assumption of evidential independence of the modalities to be fused. They are monotonic, commutative and associative functions. As t-norms are associative, fusion of three or more modalities can be done irrespective of the order. Their corresponding boundary conditions, i.e., the evaluation of the t-norms and t-conorms at the extremes of the  $[0,1]$  interval, satisfy the truth tables of the logical AND and OR operators. These norms stretch the minimum and maximum of AND and OR operators respectively thus providing a better representation of product (t-norm) and sum (t-conorm) of two variables. They are related by the DeMorgan duality, which states that if  $N(x)$  is a negation operator, then the t-conorm  $S(x, y)$  can be defined as  $S(x, y) = N(T(N(x), N(y)))$ . The multimodal fusion is carried out in [46] at the score level. After normalizing the error rates  $FAR_{ij}$  and  $FRR_{ij}$  ( $i=1$  to  $3$ ,  $j=1$  to  $n$ ) using Eq.(15) and Eq.(16), we can combine any two error rates using t-norms at a time, say  $FAR_{1j}$  and  $FAR_{2j}$  first, to yield  $T(FAR_{1j}, FAR_{2j})$ , and lastly  $FAR_{3j}$  is combined with,  $T(FAR_{1j}, FAR_{2j})$  to yield the final aggregated error rate  $T(FAR_{3j}, T(FAR_{1j}, FAR_{2j}))$ . The order of combination is immaterial due to the associative and commutative properties of the t-norms. So the final aggregated error rate is given by:

$$AFAR_j = T(FAR_{3j}, T(FAR_{1j}, FAR_{2j})) \quad (19)$$

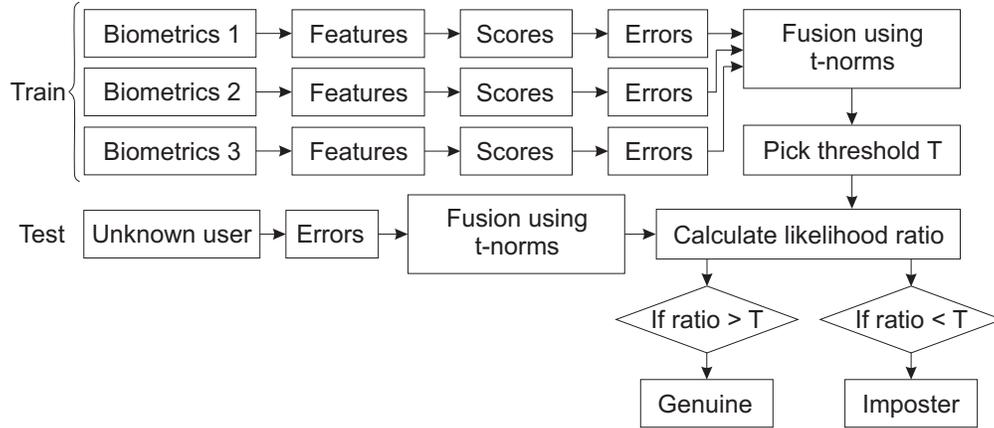
Similarly the aggregated false rejection rate is given by:

$$AFRR_j = T(FRR_{3j}, T(FRR_{1j}, FRR_{2j})) \tag{20}$$

Where  $FRR_{1j}, FRR_{2j}, FRR_{3j}$  are the normalized FRR's of three modalities. Table 1 lists out some of the t-norms. Fig.2 shows a schematic diagram of the error level fusion using t-norms.

**Table 1:** Different t-norms

t-norm	Formulation
Hamacher	$\frac{xy}{x + y - xy}$
Einstein product	$\frac{xy}{2 - x - y + xy}$
Schweizer & Sklar ( $q > 0$ )	$1 - \left( (1-x)^q + (1-y)^q - (1-x)^q(1-y)^q \right)^{1/q}$
Frank t-norm $s \in ]0, +\in[ \setminus \{1\}$	$\log_2 \left[ 1 + \frac{(s^x - 1)(s^y - 1)}{s - 1} \right]$
Yager ( $p > 0$ )	$\max \left( 0, 1 - \left( (1-x)^p(1-y)^p \right)^{1/p} \right)$



**Fig. 2:** Error level fusion using t-norms

### 3. Reinforced BF-PSO hybrid

The Choquet integral serves as the objective function in the proposed Reinforced Hybrid Learning which is a hybrid of Bacterial Foraging-Particle Swarm Optimization (BF-PSO) in the framework of Reinforcement learning (RL). The combination of PSO and BF in [29] aims at tapping the ability of PSO in exchanging the social information while tapping that of BF in finding a new solution during the elimination and dispersal step. Further, the incorporation of reinforced learning permits us to use the past information. In the BF-PSO hybrid, the velocity of the PSO becomes the direction for BF. In the hybrid the step size is one of the determining factors for the convergence of the global optima. Because of the fixed step size the hybrid algorithm has several problems [13]:

1. If the step size is very small then it requires a large number of iterations to attain the optimum solution.

2. If the step size is very high then the bacteria reach the optimum value quickly but the solution suffers from low accuracy.
3. Bacteria may get stuck up around the initial positions or the local optima, because the dispersion event happens after the specified number of reproduction processes.

These problems are removed by invoking reinforcement learning (RL) in the selection of step size. RL facilitates any algorithm to learn its behavior based on the feedback from the environment. In the case of PSO-BF hybrid, it adjusts the direction of movement of bacteria. The actions performed by an organism become feedback, which in turn is translated into a negative or positive reward for that action. When the bacterium moves to the non-favorable environment it encounters meager source of food. This negative feedback will force the bacterium to change the direction (tumble). In the case of favorable situation the bacterium ventures to move in the same direction (swim) compelled by the positive feedback. Reward of this positive feedback is the right stimulant for the optimal solution.

Reinforcement is a way to exploit the reuse policy of the past information, i.e. the error. It helps accelerate the process of exploitation. Exploitation is still possible without recourse to the reinforcement but is not definitely effective. Any policy utilizing the past information is bound to enhance the exploitation since the current information provides a local view whereas the accumulated information provides a global view. However, how best the accumulated information is made use of is the concern of this investigation. To bring the reinforcement into effect, the past information in the form of error is accumulated for each bacterium.

An exploitation mechanism through RL by way of the past information makes the updating strategies of the variables involved in PSO-BF hybrid to be adaptive. This will be elaborated now.

If  $P(i, j, k, l)$  denotes position of the  $i^{th}$  bacterium in  $j^{th}$  chemotactic step,  $k^{th}$  reproduction step, and  $ell^{th}$  elimination and dispersal step, the updating strategy of variable in PSO-BF is given by:

$$Tumble : \quad P(i, j + 1, k, l) = P(i, j, k, l) + c(i)V(i) \quad (21)$$

$$Swim : \quad P(i, j, k, l) = P(i, j, k, l) + c(i)V(i) \quad (22)$$

where  $c(i)$  is the step size for the  $i^{th}$  bacterium and  $V(i)$  is the velocity of  $i^{th}$  bacterium given by:

$$V(i) = w * V(i) + C_1 * R_1 * (P_{lbest}(i) - P_{current}(i)) + C_2 * R_2 * (P_{gbest}(i) - P_{current}(i)) \quad (23)$$

where  $P_{lbest}$  is the local best position,  $P_{gbest}$  is the global best position and  $P_{current}$  is the current position,  $C_1, C_2$ : the fixed parameters of PSO,  $R_1, R_2$ : the random numbers in PSO.

The error here is the accumulated average of the absolute difference of the objective functions in the consecutive chemotaxis steps of BF. The step size in chemotaxis steps is taken as the sigmoid function of the error. The bacteria take the step size depending on the nutrients they get hold of. This is determined by the difference of the current and the previous chemotactic steps, which leads to the converged optimum solution.

When the reinforcement concept is incorporated in PSO-BF hybrid, the equations of swim and tumble get modified to:

$$Tumble : \quad P(i, j + 1, k, l) = P(i, j, k, l) + \frac{V(i)}{1 + \exp(-error(i))} \quad (24)$$

$$Swim : \quad P(i, j + 1, k, l) = P(i, j + 1, k, l) + \frac{V(i)}{1 + \exp(-error(i))} \quad (25)$$

Where the step size  $c(i)$  is replaced by the sigmoid function of the error,

$$error(i) = \frac{\sum_j \left| J(i, j, k, ell) - J(i, j - 1, k, ell) \right|}{Total\ number\ of\ chemotaxis\ steps\ so\ far} \quad (26)$$

where the summation is taken over the chemotaxis steps.

The initial parameters belonging to BF are:  $n, S, N_c, N_s, N_{re}, N_{ed}, P_{ed}, c(i)$  ( $i = 1, 2, \dots, S$ ), and those belonging to PSO are:  $V, C_1, C_2, R_1, R_2$ . These parameters stand for  $n$ : Dimension of the search space;  $S$ : The number of bacteria in the population;  $S_r$ : Half the total number of bacteria;  $N_s$ : Maximum number of swim length;  $N_c$ : Chemotaxis steps;  $N_{re}$ : The number of reproduction steps;  $N_{ed}$ : Elimination and dispersal events;  $P_{ed}$ : probability of Elimination and dispersal;  $J(i, j, k, ell)$  is the fitness value of the  $i^{th}$  bacterium,  $j^{th}$  chemotactic step,  $k^{th}$  reproduction step, and  $ell^{th}$  elimination and dispersal event and  $J_{best}(j, k, ell)$  is the fitness of best position in  $j^{th}$  chemotaxis and  $k^{th}$  reproduction step,  $ell^{th}$  elimination and dispersal step.

#### A. Algorithm of BF-PSO Hybrid

1. Initialize the random direction  $V(i)$ , position  $P(i, 1, 1, 1)$ ,  $error(i)$  for  $i = 1 : S$
- For ( $ell = 1$  to  $N_{ed}$ )
  - For ( $k = 1$  to  $N_{re}$ )
    - For ( $j = 1$  to  $N_c$ )
      2. Evaluate the cost function  $J(i, j, k, ell)$  for  $i = 1, 2, \dots, S$
      3. Save the best cost function for  $i^{th}$  bacterium in  $J_{last}$   
 $J_{last} = J(i, j, k, ell)$
      4. The best cost for each bacterium in Step 3 is selected as the local best  $J_{local}$   
 $J_{local}(i, j) = J_{last}$
      5. Update the positions using Eq.(24)
      6. Evaluate the cost function  $J(i, j + 1, k, ell)$   
While ( $m < N_s$ ) If  $J(i, j + 1, k, ell) < J_{last}$   
 $J_{last} = J(i, j + 1, k, ell)$
      7. Update the positions using Eq.(25).
      8. Compute the cost function  $J(i, j + 1, k, ell)$
      9. Evaluate the current position and local cost of each bacterium from:  
 $P_{current}(i, J + 1) = P(i, j + 1, k, ell)$   
 $J_{local}(i, j + 1) = J_{last}$
- $m = m + 1$ ; End
- $i = i + 1$  (the next bacterium)
10. Obtain the local best position ( $P_{lbest}$ ) for each bacterium and global best position ( $P_{gbest}$ )
11. Evaluate the new direction for each  $i^{th}$  bacterium using Eq.(23).

12. Calculate the accumulated sum of absolute differences of cost functions between the consecutive chemotactic steps for each bacterium as:

$$sum(i) = \sum_j \left| J(i, j, k, ell) - J(i, j - 1, k, ell) \right|$$

13. Obtain the *error* averaged over the number of chemotaxis steps for each bacterium as:

$$error(i) = \frac{sum(i)}{no. \ of \ chemotaxis \ steps \ so \ far}$$

$j = j + 1$  (the next chemotactic step)

Evaluate  $J_{health}^i = \sum_{j=1}^{N_c+1} J(i, j, k, ell)$  and  $S_r = \frac{S}{2}$  bacteria with the highest

cost function values die and other  $S_r$  bacteria with the best values split.

$k = k + 1$  (the next reproduction step)

14. Eliminate and disperse each bacterium with probability  $P_{ed}$ .

$ell = ell + 1$  (the next elimination and dispersal step)

The parameters of reinforced hybrid learning are specified as: No. of bacteria  $S = 20$ , No. of step sizes  $N_s = 4$ , No. of chemotaxis steps  $N_c = 10$ , No. of reproduction and elimination steps  $N_{re} = 2$ , No. of elimination and dispersal steps  $N_{ed} = 2$ , the corresponding probability  $P_{ed} = 0.2$ .

#### 4. Experimental results

The error level fusion system is tested on four databases. The first database was created at I.I.T. Delhi, New Delhi. This database is prepared from the samples collected from the students and staff members of I.I.T. Delhi in the age group of 17-60 years. It consists of 100 subjects, 3 samples per subject; one each for training, validation and testing. The Gabor wavelet features [16] are obtained for the hand veins and palmprint and ICA [12] (Independent Component Analysis) features for the hand geometry. For each of the modalities, the Euclidean distance classifier is applied for delivering the scores. There are 100 genuine and 9900 imposter scores for each of the modalities. The generated genuine and imposter scores are converted into the error rates by varying the threshold with a step size of 0.001. The initial value of the threshold is set to the minimum of the genuine scores and the final value of the threshold is varied up to the maximum of the imposter scores.

The second database has emerged out of the merger of two databases, from PolyU, Hong Kong, one consisting of the knuckles [43] (PolyU Finger-Knuckle-Print Database) of both the index and middle fingers of the left hand of a person constituting two modalities and the other is palmprint [44] data which is taken as the third modality for evaluating the performance of a multimodal biometric system. This is a 'virtual' multimodal database created with the both knuckles of index and middle fingers 165 users, and the palmprints of another 165 users (PolyU palmprint database). In this we have 6 images for each modality out of which 3 for the training and 2 for the testing, and 1 for validation. The original Local Binary Pattern (LBP) operator by Ojala [39] is used for the texture analysis of palmprints and knuckles. This operator assigns a label to every pixel in an image by thresholding the gray levels of the eight neighborhood pixels with respect to the gray level of its center. The Euclidean distance is used to generate 330 (165×2) genuine scores and 54120 (165×164×2) imposter scores for each of the modalities considered.

Before using t-norms for the fusion, the error rates are normalized using Eqs.(15) and (16). The threshold that yields the highest GAR corresponding to the lowest FAR has to be selected from ROC.

Fig.3 depicts the ROCs of the individual modalities (palmprint, hand geometry, and hand veins) for the IITD database and also the ROC of the error level fusion using the Choquet integral. It may be observed that at a FAR of 0.04%, the GARs of palmprint, hand geometry and hand veins are 95%, 98% and 96% respectively. From the Fig.3, we note that with the Choquet integral, GAR is improved to 100% at FAR of 0.04%. The fuzzy densities corresponding to different thresholds are learned using the Reinforced BF-PSO hybrid.

Fig.4 shows the ROC of the error level fusion using Einstein, Hamacher, and Schweizer-Sklar t-norms on IITD Database. Einstein and Schieweizer-Sklar t-norms yield GAR of 100% at a very less FAR of  $10^{-5}\%$  and  $10^{-10}\%$  respectively. Unlike the Choquet integral that necessitates the learning of the fuzzy densities these t-norms are free from learning. The ROC in Fig.5 displays the comparison of Min, Max, weighted sum, and error level fusion using Frank t-norms on the IITD Database. From this, it is evident that Frank t-norm outperforms the rules such as Min, Max, and the weighted sum.

Table 2 gives a comparison of the authentication rates due to AND, OR rules, different t-norms and Choquet Integral for a FAR of 0.04%.

Table 3 specifies the fuzzy densities and the fuzzy measures of FAR's on IITD database.

**Table 2:** A Comparison of AND, OR, different t-norms and Choquet Integral on IITD Database.

Methodology	GAR
AND Rule	96%
OR Rule	98%
Scheweizer & Sklar t-norm (q=0.2)	100%
Einstein Product t-norm	100%
Frank t-norm (s=0.1)	100%
Choquet Integral wrt $q$ -measure	100%

**Table 3:** Fuzzy densities of FAR'S of each of the modality and the fuzzy measures on IITD database.

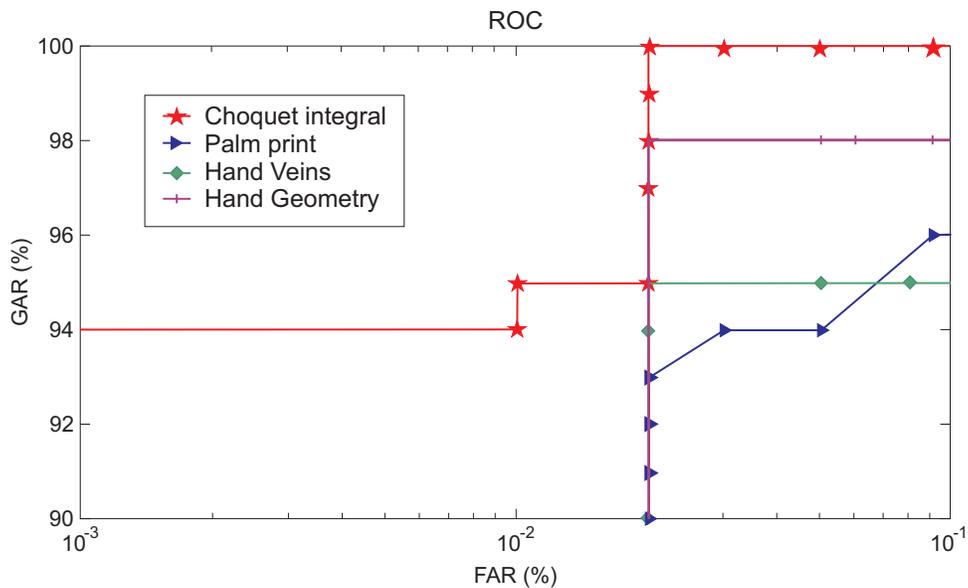
$g(\text{FAR}_1)$	$g(\text{FAR}_2)$	$g(\text{FAR}_3)$	$g(\text{FAR}_1, \text{FAR}_2)$	$g(\text{FAR}_1, \text{FAR}_2, \text{FAR}_3)$
0.3333	0.333	0.3333	0.6667	1
0.3333	0.3333	0.3333	0.6667	1
0	0	0.9952	0	1
0	0.9755	0	0.9877	1
0.3317	0.3327	0.3343	0.6648	1
0	0.2563	0.5878	0.2563	1
0	0.2563	0.5878	0.2563	1

It may be noted that higher rates of false rejection of a genuine user are caused due to AND when one of the modalities is of low quality whereas with the application of "OR" rule the false acceptance rate is found to be high even with a weak biometric. From this table it can be seen that the Choquet integral and t-norms outperform over OR and AND rules. The ROCs of individual modalities are compared with their error level fusion on PolyU database (left hand knuckles of index and middle finger, and the palmprint) using the Choquet integral. The individual authentication rates obtained with the knuckles of index and middle fingers, and the palmprint are 72%, 70%, and 82% respectively at a FAR

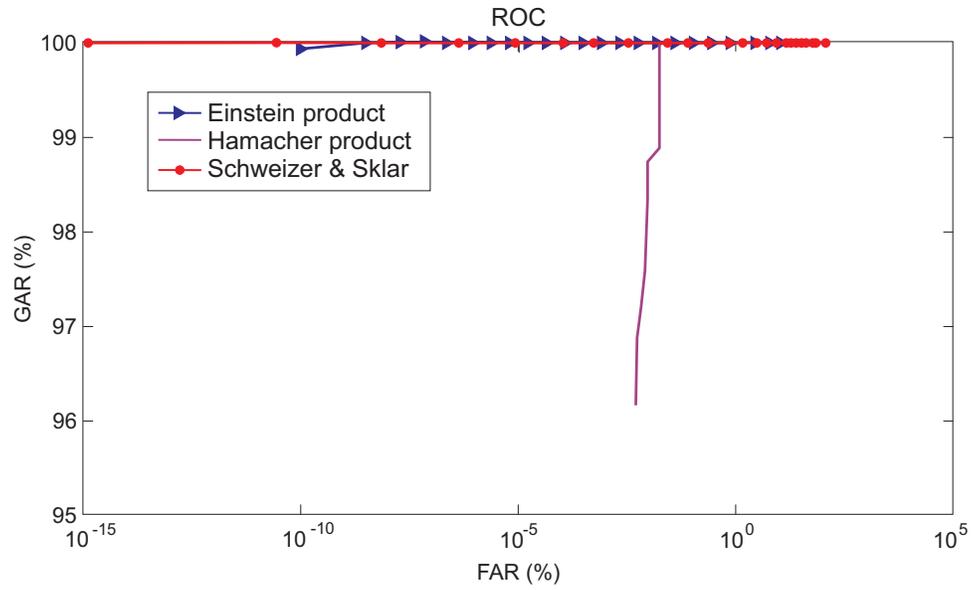
of 0.01%. However the Choquet integral gives a GAR of 99% with a FAR of  $10^{-4}$ , which is a remarkable improvement over the individual performance. Fig.7 displays the ROC's of error level fusion due to Einstein, Schweizer - Sklar, and Hamacher t-norm. With the Einstein product, GAR of 100% is obtained at FAR of  $10^{-4}$ , while with Schweizer - Sklar t-norm we achieve a GAR of 99.96% at FAR of 0.001% and with Hamacher, GAR of 99.7% at a FAR of 0.0002%.

Fig.8 shows a comparison of ROC due to OR and AND rules and Frank t-norm on PolyU Database and it is evident that Frank t-norm outweighs these rules. A comparison of the error level fusion using the Yager t-norm is made in Fig.9 with the decision level fusion due to fuzzy clustering algorithms [1] and Dempster-Shafer theory [24]. It can be observed from this figure that Yager t-norm outweighs these two methods.

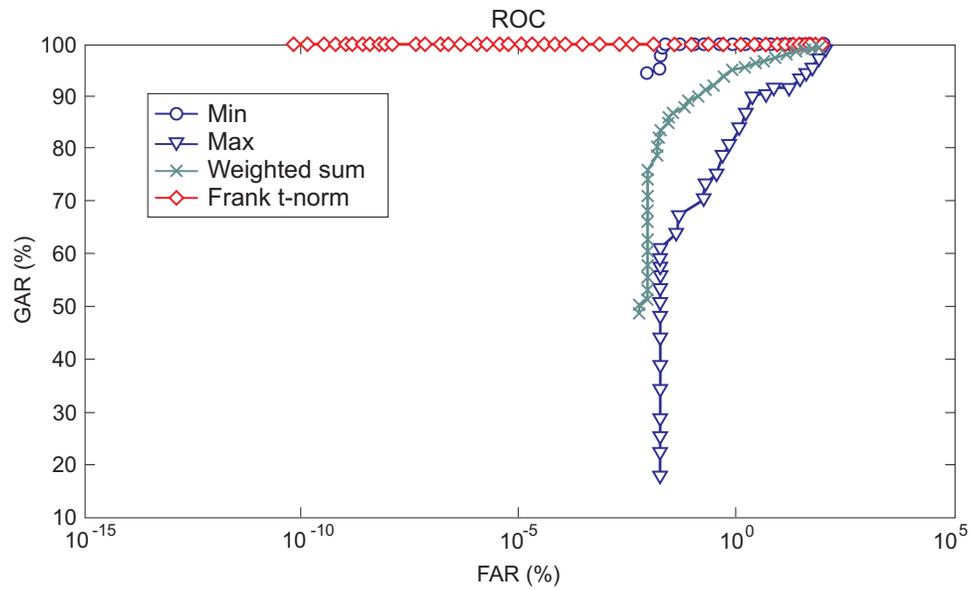
In addition to the above, two more multimodal databases, one on speech and face: XM2VTS and the other on finger and face : NIST-bssr1 are tested for the error level fusion. By employing the fusion using Frank t-norm on XM2VTS database, GAR of 99.9% is obtained at FAR of 0.001% as shown in Fig.10. For speech and face, the GARs are 87%, and 85% respectively at FAR of 0.1%. Implementation of the Choquet integral on NIST-bssr1 results in GAR of 100% at FAR of 0.002% as in Fig.11. These results are superior to what we got separately for face and finger.



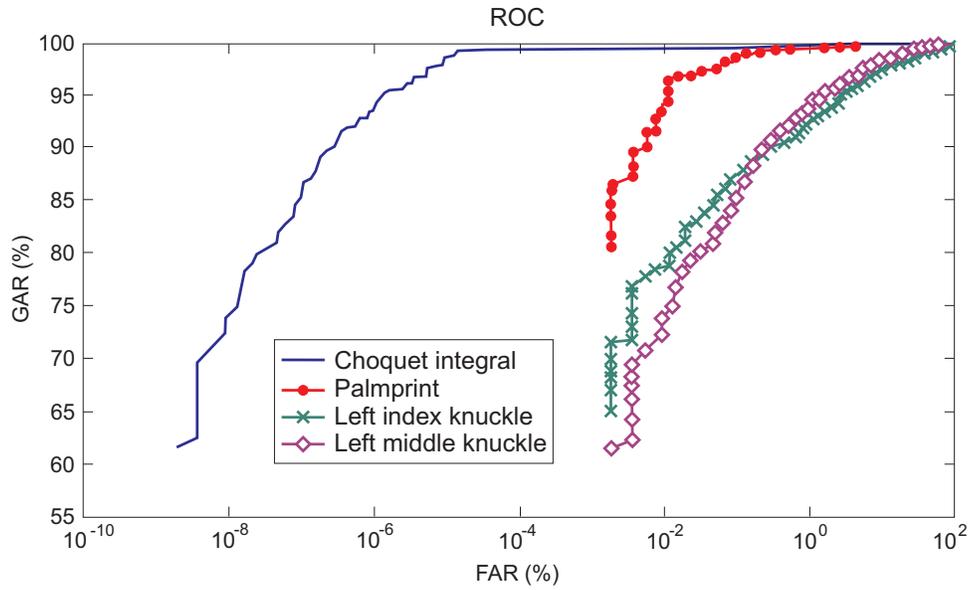
**Fig. 3:** ROCs of all the modalities ( Palmprint, Hand Geometry, and hand veins) and fusion using the Choquet Integral on IITD Database.



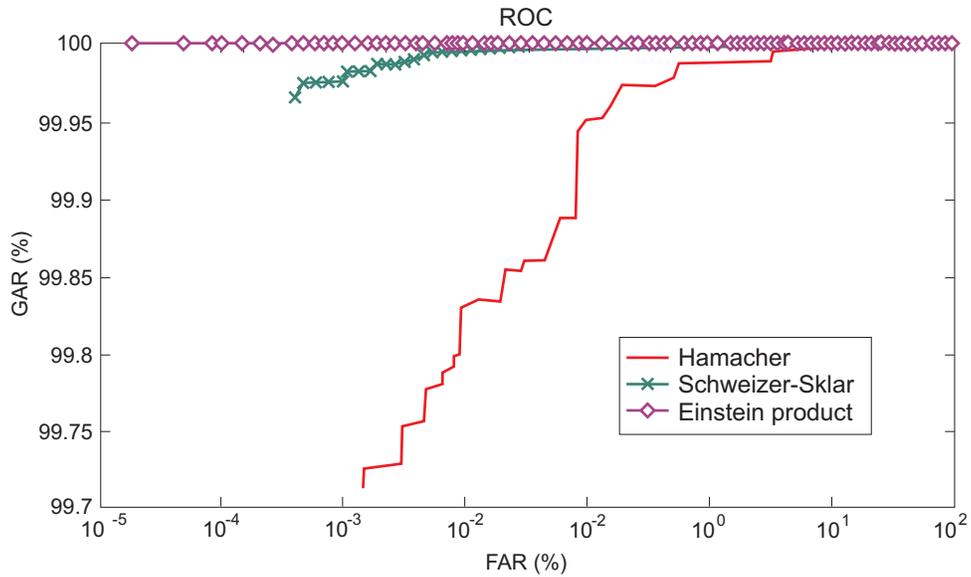
**Fig. 4:** ROC's due to the error level fusion using Einstein product, Hamacher product, and Schweizer & Sklar t-norms on IITD Database



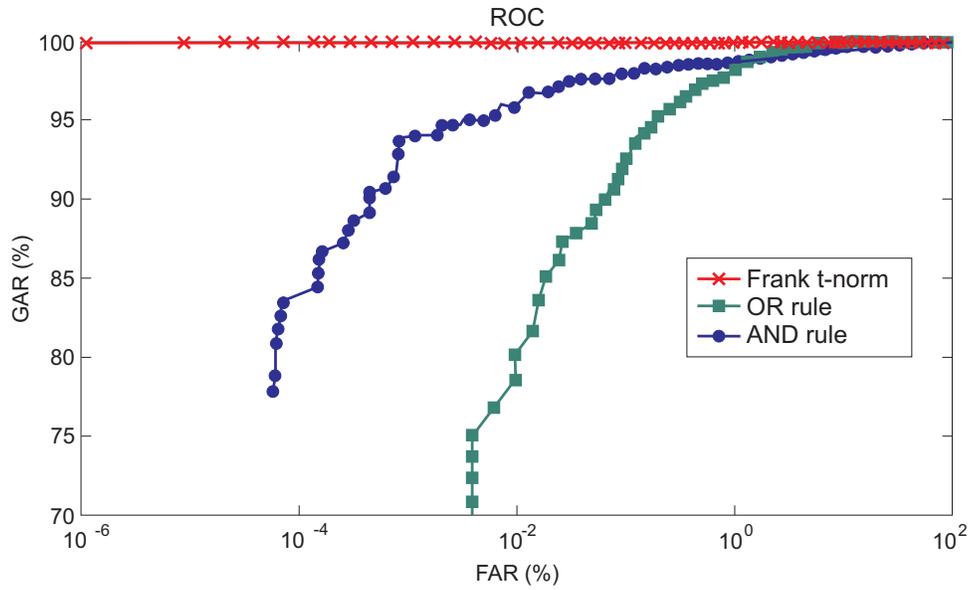
**Fig. 5:** Comparison of ROCs due to the fusion using Min, Max, Weighted sum, and error level fusion using Frank t-norm on IITD Database



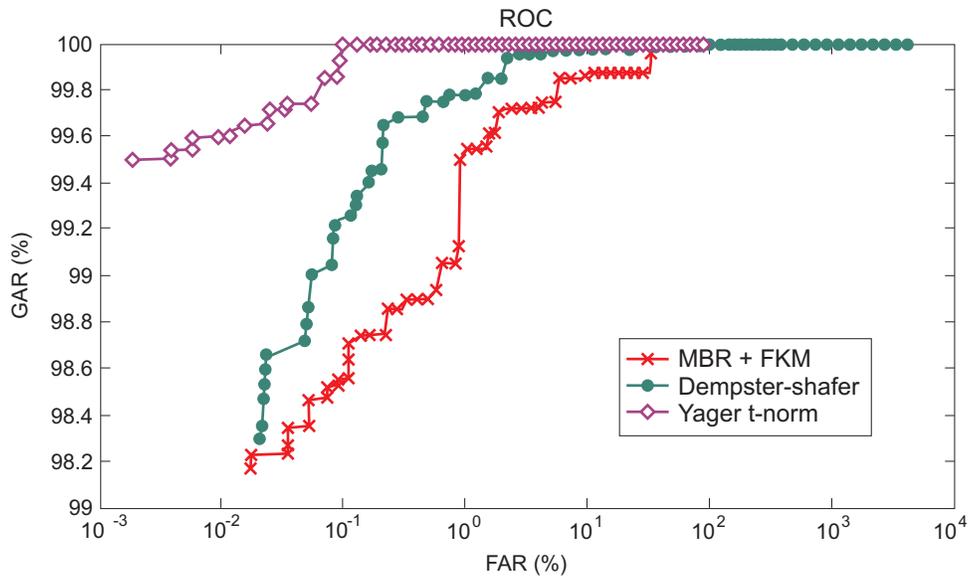
**Fig. 6:** Comparison of ROC's of all the modalities with the fusion using the Choquet integral on PolyU database



**Fig. 7:** ROC's of the fusion due to Einstein product, Hamacher product, and Schweizer & Sklar tnorm on PolyU database



**Fig. 8:** ROC's of the fusion due to Frank t-norm, OR, and AND rule on PolyU database



**Fig. 9:** A comparison of performance due to decision level fusion using MRBF + FKM, Dempster-Shafer, and the error level fusion using the Yager t-norm.

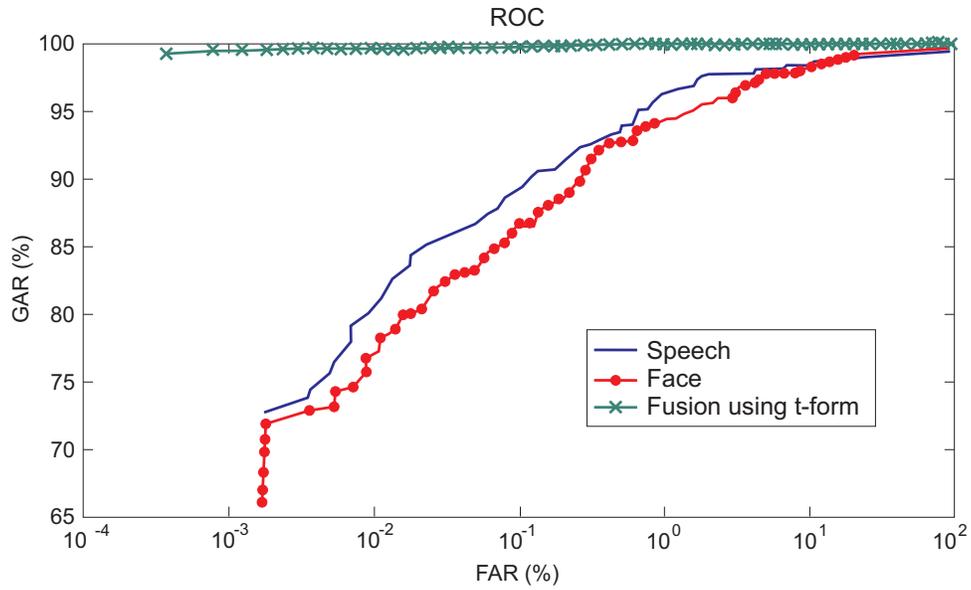


Fig. 10: Performance evaluation of Frank t-norm on XM2VTS database

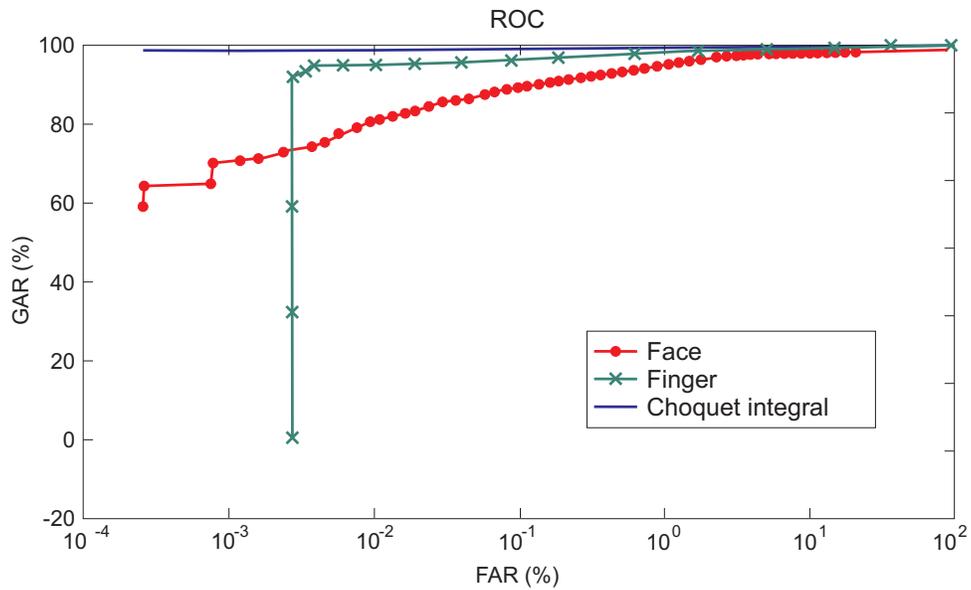


Fig. 11: Performance evaluation of error level fusion using the Choquet Integral on NIST -bssr1 database

## 5. Conclusions

The error level fusion for the multimodal biometrics is attempted here on four databases with two on hand based databases, the third on speech and face, and the fourth on finger and face. Two fusion strategies one using the Choquet integral and another using the t-norms are developed for the proposed multimodal system that is capable of accommodating any number of modalities.

The first fusion strategy deals with the fusion of the error rates of all modalities using the Choquet Integral. The use of  $q$ -measure in place of  $\lambda$ -measure drastically simplifies the computations of the interactions in the Choquet integral. The fuzzy densities learned using the Reinforced hybrid learning technique with the Choquet integral as the criterion function give the fuzzy measures of FAR and FRR of each modality for fusion. This formulation mitigates several drawbacks of the existing decision level fusion schemes. The non additive aspect of the Choquet integral is capitalized by taking care of the interaction between the error rates of two modalities at a time. Also the integral helps decide the order in which the decisions should be combined to yield better accuracy.

The second fusion strategy employs t-norms for fusion as these are overwhelmingly simpler not needing learning. Several t-norms like: Einstein, Yager, and Frank t-norm have been tried to see their relative performance in the error level fusion. Moreover t-norms are found superior to the Choquet integral performance-wise. The contributions of the paper include: the error level fusion i) using the Choquet integral that employs the  $q$ -measures, ii) using the t-norms, and iii) performance evaluation of these two fusion strategies on four databases.

The overriding factors in favor of fusion of errors derived from any type of features are i) they are compatible, ii) few in numbers, and iii) easy to fuse. The reason for error rates not being explored for fusion could be the lack of information content in them as they are a form of decisions only. The present work is a bold step to propagate this line of fusion.

## Acknowledgments

We are grateful to the Ministry of Communication & Information Technology, India for the funding provided to undertake the Multimodal Biometric Fusion at IIT Delhi.

## References

- [1] V. Chatzis, A. G. Bors, and I. Pitas, "Multimodal Decision- Level Fusion for Person Authentication," *IEEE Trans. Systems, Man, and Cybernetics*, vol. 29, no. 6, pp. 674-680, Nov. 1999.
- [2] K. Veeramachaneni, L. Osadciw, A. Ross, and N. Srinivas, " Decision-level Fusion Strategies for Correlated Biometric Classifiers," *Proc. of IEEE Computer Society Workshop on Biometrics at the Computer Vision and Pattern Recognition (CVPR) conference*, Anchorage, AK, USA, pp. 1-6, June 2008.
- [3] Q. Tao, and R. Veldhuis, "Threshold-optimized decision-level fusion and its application to biometrics," *Pattern Recognition*, vol. 42, no. 5, pp. 823-836, May 2009.
- [4] Murofushi and M. Sugeno, "A Theory of Fuzzy Measures: Representations, Choquet integral and null sets," *J. Math. Anal. Appl.*, vol. 159, no. 2, pp. 532-549, 1991.
- [5] A. Kumar, D. C. M. Wong, H. C. Shen, and A. K. Jain, "Personal Verification Using Palmprint and Hand Geometry Biometric," *Proc. of Fourth International Conference on Audio- and Video-Based Biometric Person Authentication (AVBPA)*, Guildford, U.K, pp. 668-678, June 2003.
- [6] A. Ross and R. Govindarajan, "Feature Level Fusion Using Hand and Face Biometrics," *Proc. of SPIE Conference on Biometric Technology for Human Identification, Florida, U.S.A.*, vol. 5779, pp. 196-204, Mar. 2005.
- [7] Zhou, and B. Bhanu, "Feature fusion of side face and gait for video-based human identification," *Pattern Recognition*, vol. 41, no. 5, pp. 778-795, Mar. 2008.
- [8] Y. Gao, and M. Maggs, "Feature-Level Fusion in Personal Identification," *Proc. of the 2005 IEEE Computer Society Conference on Computer Vision and Pattern Recognition (CVPR'05)*, vol. 1, San Diego, CA, USA, pp. 468 - 473, June 2005.
- [9] Y F Yao, X Y Jing, and H S Wong, "Face and palmprint feature level fusion for single sample biometrics recognition," *Neurocomputing*, vol. 70, no. 7, pp. 1582-1586, 2007 .
- [10] A. Rattani, D. R. Kisku, M. Bicego, "Feature Level Fusion of Face and Fingerprint Biometrics," *Proc. of Biometrics: Theory, Applications, and Systems (BTAS)*, Washington, DC, pp. 27-29, Sep. 2007.
- [11] K. Nandakumar, *Integration of Multiple Cues in Biometric Systems*, M.S. Thesis, University of Michigan, 2005.
- [12] P. Comon (1994), "Independent Component Analysis: a new concept?," *Signal Processing, Elsevier*, vol. 36, no. 3 pp. 287-314, 1994.
- [13] Datta T, Misra I. S., "Improved adaptive bacteria foraging Algorithm in optimization of antenna array for faster convergence" , *Electromagnetic Research C*, vol. 1, pp.143-157, 2008.
- [14] S. C. Dass, K. Nandakumar, and A. K. Jain. "A Principled Approach to Score Level Fusion in Multimodal Biometric Systems," *In Fifth AVBPA*, Rye Brook, NY, USA, pp. 1049- 1058, July 2005.
- [15] Chih-Lung Lin, and Kuo-Chin Fan, "Biometric verification using thermal images of palm-dorsa vein patterns", *IEEE Trans. on Circuits and Systems for Video Technology*, vol. 14, no. 2, pp. 199-213, 2004.
- [16] A. Kumar, M. Hanmandlu, and H. M. Gupta, "Online Biometric Authentication Using Hand Vein Patterns," *Proceedings of the 2009 IEEE Symposium on Computational Intelligence in Security and Defense Applications (CISDA 2009)*, Ottawa, Ontario, Canada, pp. 311-317, 2009.
- [17] M. S. Fahmy, A. F. Atyia, Raafat , S. Elfouly, "Biometric Fusion Using Enhanced SVM Classification," *Fourth International Conference on Intelligent Information Hiding and Multimedia Signal Processing*, Harbin, China, pp. 1043-1048, Aug. 2008.
- [18] Hyun-Ae Park, Kang Ryoung Park, "Iris recognition based on score level fusion by using SVM," *Pattern Recognition Letters*, vol. 28, no. 15, pp. 2019-2028, 2007.
- [19] A. Kumar, and D. Zhang, "User Authentication Using Fusion of Face and Palmprint," *International journal of Image and Graphics*, vol. 9, no. 2, pp. 251-270, 2009.

- [20] K. Nandakumar, Y. Chen, S. Dass, A. Jain, "Likelihood ratio-based biometric score fusion," *IEEE Trans. Pattern Anal. Mach. Intell.*, vol. 30, no. 2, pp. 342-347, 2008.
- [21] L. Lam and Ching Y. Suen, "Application of Majority Voting to Pattern Recognition: An Analysis of Its Behavior and Performance," *IEEE Tran. on Systems, Man, And Cybernetics*, vol. 27, no. 5, pp. 553-568, 1997 .
- [22] R. C. Gonzalez, R. E. Woods, *Digital Image Processing*, Addison Wesley, publishers, 1993.
- [23] Hakan Altunay, "On naive Bayesian fusion of dependent classifiers," *Pattern Recognition Letters*, vol. 26, no. 15, pp. 2463-2473, 2005.
- [24] A. A. Ani, and M. Deriche, "A new technique for combining Multiple classifiers using the Dempster-Shafer theory of evidence," *Journal of Artificial Intelligence Research*, vol. 17, no. 1, pp. 333-361, 2002.
- [25] J. Daugman, Combining multiple biometrics, [www.cl.cam.ac.uk/~jgd1000/combine/combine.html](http://www.cl.cam.ac.uk/~jgd1000/combine/combine.html).
- [26] Sansanee Auephanwiriyakul, James M. Keller, and Paul D. Gader, "Generalized Choquet fuzzy integral fusion," *Information Fusion*, vol. 3, no. 1, pp. 69-85, 2002.
- [27] X. Chen, Z. Jing, and G. Xiao, "Nonlinear fusion for face recognition using Fuzzy integral," *Communications in Nonlinear Science and Numerical Simulation*, vol. 12, no. 5, pp. 823- 831, 2007.
- [28] M. Grabisch, "A new algorithm for identifying fuzzy measures and its application to pattern recognition," *Fourth IEEE International Conference on Fuzzy Systems*, Yokohama, Japan, pp. 145-150, Mar. 1995.
- [29] Wael Mansour Korani, "Bacterial foraging oriented by particle swarm optimization strategy for PID tuning," *Proceedings of the 2008 GECCO conference companion on Genetic and evolutionary computation*, Atlanta, GA, USA ,pp. 1823- 1826, July 2008.
- [30] K. Veeramachaneni, L. A. Osadciw, and P. K. Varshney, "An adaptive multimodal biometric management Algorithm," *IEEE Trans. Syst., Man, Cybern. C, Appl. Rev.*, vol. 35, no. 3, pp. 344-356, Aug. 2005.
- [31] J. Daugman, "Complete Discrete 2-D Gabor Transforms by Neural Networks for Image Analysis and Compression," *IEEE Trans. on Acoustics, Speech, and Signal Processing*, vol. 36, no. 7, pp. 1169-1179, July 1988.
- [32] V. Novák, W. Pedrycz, "Fuzzy sets and t-norms in the light of fuzzy logic", *International Journal of Man-Machine Studies*, vol. 22, no. 2, pp. 113-127, 1988.
- [33] K. Atoh, and Wei-Yun Yau, "Combination of Hyperbolic Functions for Multimodal Biometric Fusion," *IEEE Trans. on Systems, Man, and Cybernetics*, Part B, vol.34, no.2, pp. 1196-1209, 2004.
- [34] Klement, Erich Peter; Mesiar, Radko; and Pap, Endre (2000), *Triangular Norms*. Dordrecht: Kluwer. ISBN 0-7923-6416-3 .
- [35] Magdi A. Mohamed, and Weimin Xiao, "Q-Measures: An Efficient Extension of the Sugeno -Measure," *IEEE Trans. Fuzzy Systems*, vol. 11, no. 3, pp. 419-426, June 2003.
- [36] Salil Prabhakar, and A. K. Jain, "Decision-level fusion in fingerprint verification," *Pattern Recognition*, vol. 35, no. 4, pp. 861-874, 2002.
- [37] Dae Jong Lee, Keun Chang Kwak, Jun Oh Min, and Myung Geun Chun, " Multimodal Biometrics System Using Face and Signature," *LNCS*, vol. 3043 , pp. 635-644, 2004.
- [38] David Zhang, Wai-Kin Kong, Jane You, and Michael Wong, "Online Palmprint Identification", *IEEE Trans. Pattern Analysis & Machine Intelligence*, vol. 25, no. 9, pp. 1041 - 1050, 2003.
- [39] T. Ojala, M. Pietikäinen, and T. Mäenpää, "Multiresolution Gray-Scale and Rotation Invariant Texture Classification with Local Binary Patterns", *IEEE Trans. on Pattern Analysis and Machine Intelligence*, vol. 24, pp. 469-481, July 2002.
- [40] Ludmila I. Kuncheva,, "Theoretical Study on classifier level fusion Strategies," *IEEE Trans. on Pattern Analysis and Machine Intelligence*, vol. 24, no. 2, pp. 281-286, 2002.
- [41] Hui-Min Feng, Xue-Fei Li; Jun-Fen Chen, "A comparative study of four fuzzy integrals for classifier fusion," *2010 International Conference on Machine Learning and Cybernetics (ICMLC)*, Qingdao, pp. 332-338, July 2010.
- [42] N. Littlestone, M. Warmuth, "The weighted majority algorithm," *Inf. Comput.*, vol. 108, no. 2, pp. 212-261, 1994.

- [43] Lin Zhang, Lei Zhang, David Zhang, and Hailong Zhu, "Online finger-knuckle-print verification for personal authentication," *Pattern Recognition*, vol. 43, no. 7, pp. 2560-2571, July 2010.
- [44] David Zhang, Wai-Kin Kong, Jane You and Michael Wong, "On-line palmprint identification", *IEEE Transactions on Pattern Analysis and Machine Intelligence*, vol. 25, no. 9, pp. 1041-1050, 2003.
- [45] Hanmandlu Madasu, Grover Jyotsana, Vamsi Krishna Madasu, "Decision Level Fusion Using t-Norms", *2010 International Conference on Digital Image Computing: Techniques and Applications (DICTA)*, Sydney, NSW, pp. 33-38, Dec. 2010.
- [46] Madasu Hanmandlu, Jyotsana Grover, Ankit Gureja , and H.M. Gupta, "Score Level Fusion of Multimodal Biometrics Using Triangular Norms," *Pattern Recognition Letters* (in press).
- [47] S. M. Kay. *Fundamentals of Statistical Signal Processing: Detection Theory*. Englewood Cliffs, NJ: Prentice-Hall, vol. II, pp. 1176-1183, 1998.